

CEBÂȘEV - TYPE MECHANISMS OBTAINED THROUGH THE OPTIMUM SYNTHESIS BASED ON SOME CALCULUS PROGRAMS

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Abstract: In the present paper we render a few numerical applicative cases of a general method for optimum synthesis of Cebâșev mechanisms [4] having on its basis calculus algorithms and programs proposed by the authors. We achieve the optimum synthesis of Cebâșev type mechanisms, in which the trajectory of a point on a piston rod approximates a circle arch or a straight line.

According to optimizing criteria, based on the calculus algorithms and programs, we obtained a large range of plane quadrilateral mechanisms which can be selected according to the working field.

Keywords: synthesis method, Cebâșev mechanisms, numerical calculus method.

INTRODUCTION

In the paper no. [4], we present the theoretical aspects of a general method for the optimum synthesis of Cebâșev mechanisms. In the present paper, a number of numerical cases are solved, on the basis of algorithm and calculus program for the optimum synthesis of Cebâșev mechanisms.

THEORETICAL NOTIONS AND CALCULUS ALGORITHM

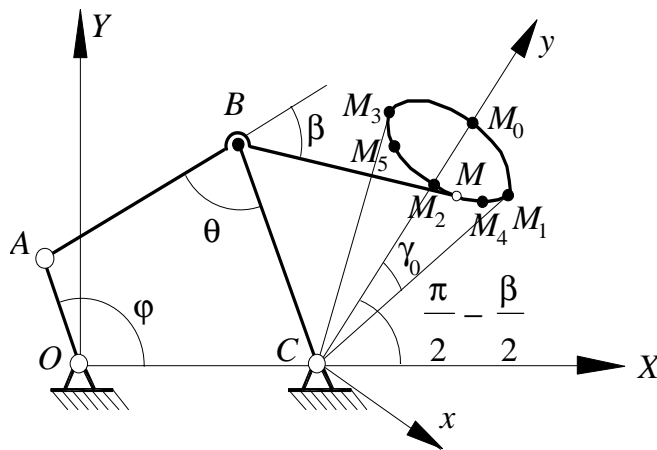


Fig. 1

We consider Cebâșev mechanism in the 1st figure, in which we note $OA = a$; $OC = d$; $AB = BC = BM = 1$, $\lambda = \frac{d}{a}$. And we ask to determine the parameters λ , a , β that point M describe the arc of curve M_4, M_5 which approximates a segment of a straight line parallel to axis Cx

We consider that points M_4, M_5 correspond to the values of the angle φ equal to $90^\circ, 270^\circ$.

The synthesis has at its basis, relations established in the paper no.[4], that is:

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$$\theta = 2 \arcsin \left(\frac{a}{2} \sqrt{1 + \lambda^2 - 2\lambda \cos \varphi} \right) \quad (1)$$

$$\gamma = \arcsin \left(\frac{\sin \varphi}{\sqrt{1 + \lambda^2 - 2\lambda \cos \varphi}} \right) \quad (2)$$

$$x = 2 \cos \frac{\theta + \beta}{2} \sin \gamma \quad (3)$$

$$y = 2 \cos \frac{\theta + \beta}{2} \cos \gamma \quad (4)$$

Then, given the angle φ the values we determine the parameter θ_i, γ_i, y_i , based on relations (1) – (4) and then the average:

$$\bar{y} = \frac{\sum_{i=1}^{90} y_i}{90} \quad (5)$$

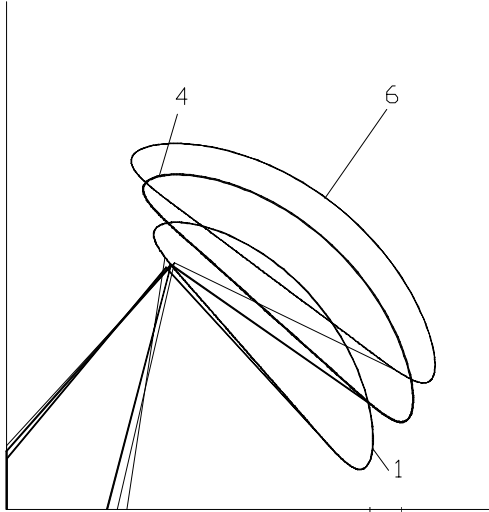


Fig. 2

To determine the minimal value of the function objective.

$$S = \frac{1}{x_1} \sum_{i=1}^{90} |y_i - \bar{y}| \quad (6)$$

where

$$x_1 = 2 \cos \frac{\theta_1 + \beta}{2} \sin \gamma_1 \quad (7)$$

taking into account the limitations [4] so that the mechanism be of the swing support type:

$$a < 1; a(\lambda + 1) < 2 \quad (8)$$

(To determine the optimum mechanisms a calculus

program in Pascal has been initiated).

We consider for a parameter, values hundredth to hundredth, starting from the value $a = 0,2$ and ending with value $a = 0,69$. Analogous, we consider for λ variable values from hundredth to hundredth starting with $\lambda = 1,1$ up to a value $\lambda = 11$ and for the β we consider values changing gradually starting with value $\beta = 0^\circ$ and ending with value $\beta = 170^\circ$.

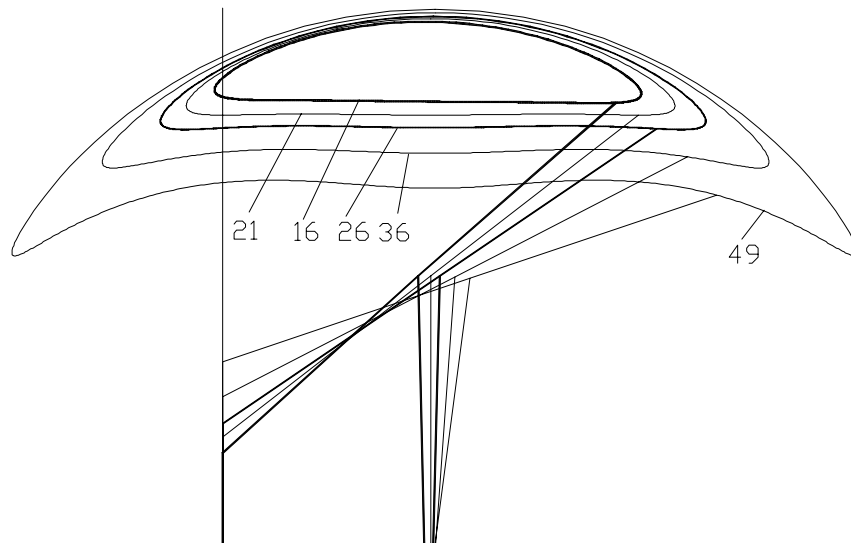


Fig.3

For every value given to the parameter a , we hold back the mechanism for which the function objective is minimum.

Out of the analysis we draw the first conclusion that is the deviation related to the segment of the straight line belonging to the point M' trajectory increases in the same time the length of the crank increases.

We also see that at small values of the crank's length the optimum mechanisms have got bigger values for the angle β and for bigger values of the crank's length, the optimum mechanisms are obtained for the value 0 of the angle β .

These things are emphasized in the diagrams of the mechanisms' trajectories in fig.2 and fig.3

THEORETICAL CONCEPTS AND CALCULUS ALGORITHM FOR OPTIMUM SYNTHESIS OF CEBÂSEV MECHANISMS IN WHICH THE TRAJECTORY OF A POINT ON THE PISTON RED APPROXIMATES A CIRCLE' ARCH

This synthesis, for $\varphi \in [-\tilde{\varphi}, \tilde{\varphi}]$ where takes the maximum value of 70° has at its basis paper [4] with the following relations:

$$\theta_0 = 2 \arcsin\left(\frac{a}{2}\sqrt{\lambda-1}\right) \quad (9)$$

$$\rho_0 = 2 \cos \frac{\theta_0 + \beta}{2} ; \quad [\tilde{\varphi}] = N ; \quad (10)$$

$$\tilde{\theta} = 2 \arcsin\left(\frac{a}{2}\sqrt{1+\lambda^2 - 2\lambda \cos N^\circ}\right) ; \quad (11)$$

$$\tilde{\gamma} = \arcsin\left(\frac{\sin N^\circ}{\sqrt{1+\lambda^2 - 2\lambda \cos N^\circ}}\right) ; \quad (12)$$

$$\tilde{\rho} = 2 \cos \frac{\tilde{\theta} + \beta}{2} ; \quad \tilde{y} = \tilde{\rho} \cos \tilde{\gamma} ; \quad (13)$$

$$R = \frac{\tilde{\rho}^2 + \rho_0^2 - 2\tilde{y}\rho_0}{2(\rho_0 - \tilde{y})} \quad (14)$$

If $\varphi_i = i - 1$, then we obtain the relation:

$$\theta_i = 2 \arcsin\left(\frac{a}{2}\sqrt{1+\lambda^2 - 2\lambda \cos \varphi_i}\right) \quad (15)$$

$$\gamma_i = \arcsin\left(\frac{\sin \varphi_i}{\sqrt{1+\lambda^2 - 2\lambda \cos \varphi_i}}\right) \quad (16)$$

$$\rho_i = 2 \cos \frac{\theta_i + \beta}{2} \quad (17)$$

$$x_i = \rho_i \sin \gamma_i ; \quad y_i = \rho_i \cos \gamma_i \quad (18)$$

$$S = \frac{1}{|R|} \sum_{i=1}^N \left| \sqrt{x_i^2 + (y_i + \rho_0 + R)^2} - |R| \right| \quad (19)$$

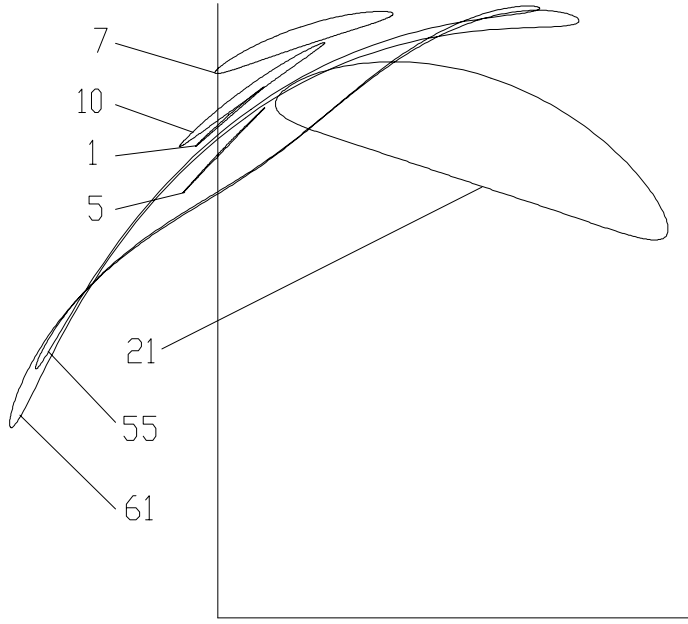


Fig. 4

Based upon the above mentioned relation the following calculus program has been created in Pascal.

For the case when $\tilde{\varphi} = 40^\circ$ we obtain based on the program, the mechanisms having the sizes from fig.4 and for $\tilde{\varphi} = 70^\circ$ the mechanisms of fig.5.

It comes out that the optimum solutions are for $\beta = 0$

The synthesis for $\varphi \in [\tilde{\varphi}, 360^\circ - \tilde{\varphi}]$, where $\tilde{\varphi}$ takes as a maximum value 100° , has at the base paper [4] the following relation:

$$\theta_\pi = 2 \arcsin \frac{a|\lambda + 1|}{2}; \quad (20)$$

$$\rho_\pi = 2 \cos \frac{\theta_\pi + \beta}{2}; \quad (21)$$

were: $[\tilde{\varphi}] = N, :$

$$\tilde{\theta} = 2 \arcsin \left(\frac{a}{2} \sqrt{1 + \lambda^2 - 2\lambda \cos N^\circ} \right); \quad (23)$$

$$\tilde{\gamma} = \arcsin \left(\frac{\sin N^\circ}{\sqrt{1 + \lambda^2 - 2\lambda \cos N^\circ}} \right); \quad (24)$$

$$\tilde{\rho} = 2 \cos \frac{\tilde{\theta} + \beta}{2}; \quad (25)$$

$$\tilde{y} = \tilde{\rho} \cos \tilde{\gamma} \quad (26)$$

$$R = \frac{\tilde{\rho}^2 + \rho_0^2 - 2\tilde{y}\rho_0}{2(\rho_0 - \tilde{y})}; \quad (27)$$

When $\varphi_i = N - 1 + i$, we obtain the relation:

$$\theta_i = 2 \arcsin \left(\frac{a}{2} \sqrt{1 + \lambda^2 - 2\lambda \cos \varphi_i} \right); \quad (28)$$

$$\gamma_i = \arcsin \left(\frac{\sin \varphi_i}{\sqrt{1 + \lambda^2 - 2\lambda \cos \varphi_i}} \right); \quad (29)$$

$$\rho_i = 2 \cos \frac{\theta_i + \beta}{2}; \quad (30)$$

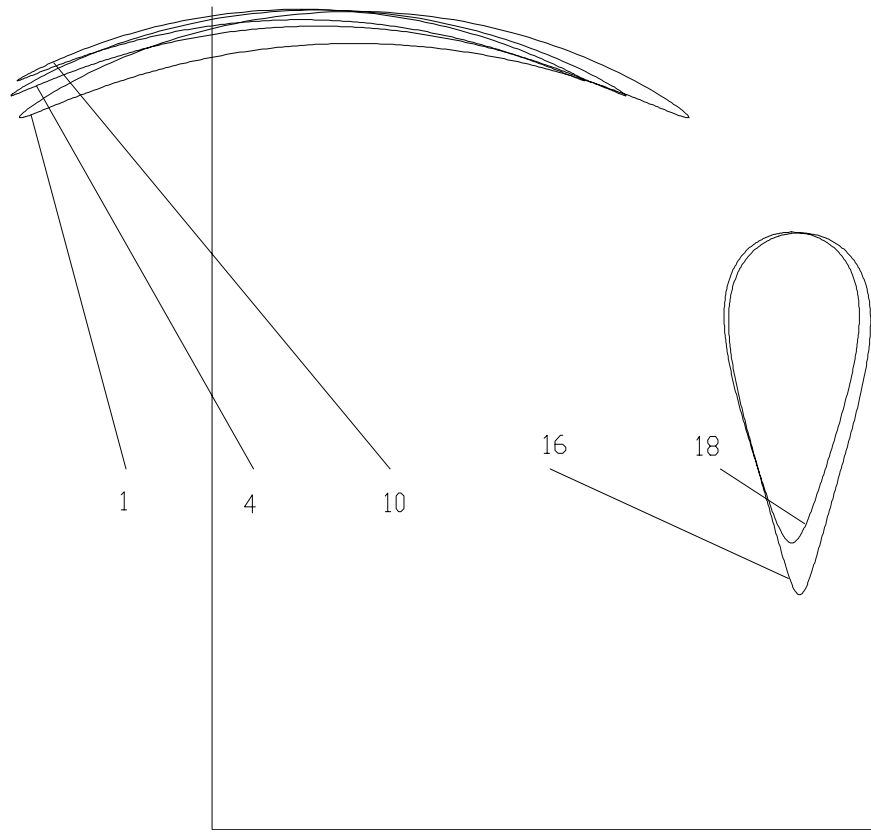


Fig. 5

$$x_i = \rho_i \sin \gamma_i; \quad y_i = \rho_i \cos \gamma_i; \quad (31)$$

$$S = \frac{1}{|R|} \sum_{i=1}^{180-N+1} \left| \sqrt{x_i^2 + (y_i + \rho_0 + R)^2} - |R| \right|; \quad (32)$$

Similarly, a calculus program has been made, whose listing, for shortage of space cannot be displayed. Based on this one, we can conclude that optimum solutions in case $\tilde{\varphi} = 100^0$ lead to circle' arches with a large radius, when cases in which circle arches may be assimilated to straight-line segments. In fig.6 we show the mechanisms and the trajectory described by point M in on the piston rod for some of the optimum solutions.

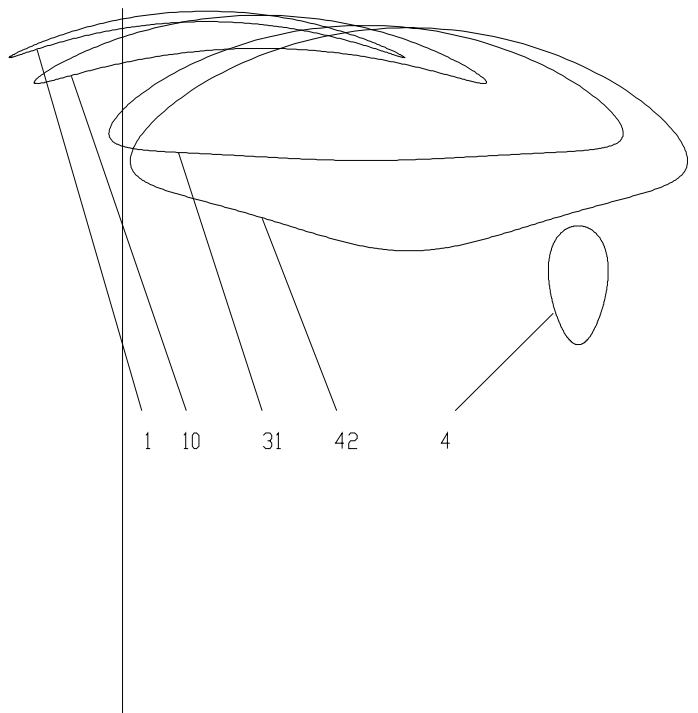


Fig. 6

CONCLUSIONS

According to the optimization criteria used, in the paper there are displayed types of such optimum mechanisms, obtained due to calculus programs.

These calculus programs generate a variety of mechanisms that will be selected according to the working field.

The method used in the paper allows based on the calculus programs, obtaining through optimum synthesis of Cebâșev mechanisms.

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