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# THEORETICAL ASPECTS REGARDING PLANE QUADRILATERAL MECHANISMS OBTAINED THROUGH OPTIMUM SYNTHESIS

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***Abstract.*** *The paper presents a general method for the optimum synthesis of Cebâsev type plane quadrilateral mechanisms, methods based on position functions and numerical calculus method. By the suggested method we make optimum synthesis to generate a trajectory of a point on the position rod' plane which approximates a straight line or a circle arch, obtaining different mechanisms. Numerical applications of the method based on calculus algorithms and programmes are rendered in paper [5].*

**Keywords:** synthesis method, Cebâsev mechanisms, numerical calculus method.

## INTRODUCTION

Cebâsev mechanisms are articulate quadrilateral plane mechanisms, built that a certain point of the piston rod make approximately lither a circle arch or a straight line segment

These mechanisms are, generally speaking of the type – crank swing support and is characterized by the fact the lengths of the piston rod, of the swing support and of the distance between generating point on the joint piston rod – swing support are equal. For the synthesis of such mechanisms we appeal [4] to Cebâsev analytical method and through analytic laborious calculus we fond Artobolevski mechanisms presented by this one in the 1st volume of the paper [1].

In the paper we elaborate a general method for the synthesis of Cebâsev – type mechanisms, method based on the position function and methods of numerical calculus.

## GENERAL EQUATIONS

We consider Cebâsev quadrilateral mechanisms in fig.1 related to a system of axes OXY and we use notations:

$$A = a;\ OC = d;\ AB = BC = BM = b \tag{1}$$

where: $\varphi$ - angle between crank $OA$ and axis $OX$ ; $\theta$ – angle between piston rod $AB$ and swing support $BC$; $\gamma$ – angle between straight lines $CA$, $CO$; $\beta$ - angle between segments $BM$ and piston rod $AB$.

In the position $\varphi = 0$ (fig. 2) the $CM_0$ slants with an angle $\frac{\pi}{2}-\frac{\beta}{2}$ towards the axis $OX$ and the angles $\angle ACO$, $\angle MCM_0$ (fig.1) are equal and have the same measure $\gamma$.

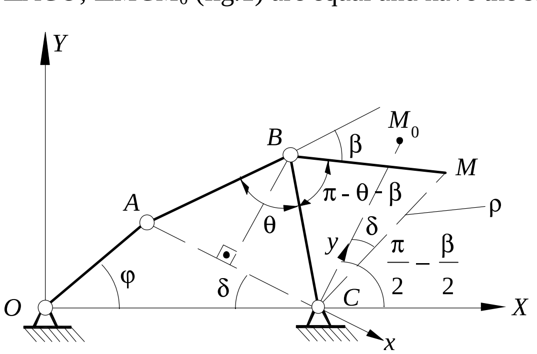


Fig. 1

Under these circumstances, the position of point $M$ related to a system of axes $Cxy$ (fig.1) where the axis $Cy$ coincides with straight line $CM_0$ slanting with an angle $\frac{\pi}{2}-\frac{\beta}{2}$ toward the axis $OX$.

As the triangles $ABC$, $CBM$ are isosceles and noting $\rho$ the polar distance $CM$, it results the equalities:

$$AC = 2b\sin\frac{\theta}{2} \tag{3}$$

$$\rho = 2b\cos\frac{\theta+\beta}{2} \tag{4}$$

Applying the theorem of the cosine in the triangle *ACO* we infer the equality:

$$AC = \sqrt{a^2 + d^2 - 2ad\cos\varphi}\,; \tag{5}$$

and taking into account the relation (3) we infer the expression:

$$\sin\frac{\theta}{2} = \frac{1}{2b}\sqrt{a^2 + d^2 - 2ad\cos\varphi} \tag{6}$$

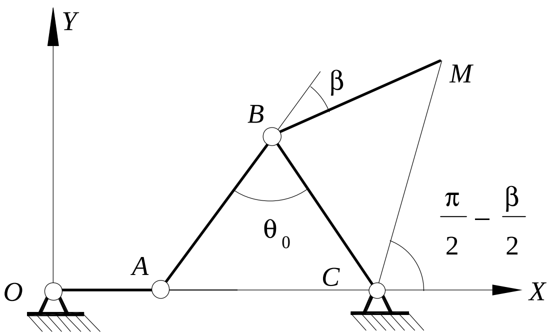

Fig. 2.

from which it comes out:

$$\theta = 2\arcsin\left(\frac{1}{2b}\sqrt{a^2 + d^2 - 2ad\cos\varphi}\right) \tag{7}$$

from the sine theorem in the triangle OAC

$$\frac{OA}{\sin\gamma} = \frac{AC}{\sin\varphi}\,; \tag{8}$$

we infer with the help of relation (5) the equality:

$$\gamma = \arcsin\left(\frac{a\sin\varphi}{\sqrt{a^2 + d^2 - 2ad\cos\varphi}}\right); \tag{9}$$

Based on the relations (4) (7), (9) we infer the parametric equations of the point *M*' trajectory versus the system of axes *Cxy*, the parameter being the angle φ

$$\begin{array}{l} x = 2b\cos\dfrac{\theta+\beta}{2}\sin\gamma \\ y = 2b\cos\dfrac{\theta+\beta}{2}\cos\gamma \end{array}; \tag{10}$$

## PROPERTIES OF THE POINT'S TRAJECTORY ON THE PISTON ROD

The equations (10) show that the trajectory described by the point *M* is a closed curve and symmetric versus the axis *Cy*

This characteristic comes out of the fact that if we replace in these equations, the angle φ with $-\varphi$ then taking into account the relations (6), (9) it infers the angle θ doesn't modify and the angle γ becomes $-\gamma$ and consequently the coordinate *x* becomes $-x$ and the coordinate *y* stays invariable.

The relations (7), (9) being adimensional we can look for Cebâsev mechanisms discharging the condition *b=1*

For all the mechanisms similar to these, the resemblance report being equal to *b*, the trajectories described by point *M* will be the same

Under these circumstances if we add the notation

$$d = \lambda a \tag{11}$$

we obtain from (6), (9), (10) the following synthesis relations

$$\theta = 2\arcsin\left(\frac{a}{2}\sqrt{1+\lambda^2 - 2\lambda\cos\varphi}\right) \tag{12}$$

$$\gamma = \arcsin\left(\frac{\sin\varphi}{\sqrt{1+\lambda^2 - 2\lambda\cos\varphi}}\right) \tag{13}$$

$$x = 2\cos\frac{\theta+\beta}{2}\sin\gamma;\ y = 2\cos\frac{\theta+\beta}{2}\cos\gamma \tag{14}$$

Relation (12) shows that the angle θ takes extreme values for φ = 0, π and particularly:

$$\varphi = 0 \rightarrow \theta_{\min} = 2\arcsin\frac{a|\lambda-1|}{2} \tag{15}$$

$$\varphi = \pi \rightarrow \theta_{\max} = 2\arcsin\frac{a|\lambda+1|}{2} \tag{16}$$

If we consider the mechanism to be of crank – swift support type then the element *OA* must be the smallest, by thus implying inequalities:

$$\lambda < 1;\ 0 < a < 1 \tag{17}$$

Moreover it is necessary for angle $\theta_{max}$ to exist, fact that leads to the condition:

$$a(\lambda+1) < 2 \tag{18}$$

Cebâsev Mechanism double crank is obtained if the element OC is the smallest and further on the angle θ exists, that is when:

$$0 < \lambda < 1;\ a(\lambda+1) < 2 \tag{19}$$

In the following paragraphs we will be studying only crank-swing support mechanisms that is, those mechanisms fulfilling the conditions (17), (18).
For these mechanisms, out of the relations (4), (15), (16) comes out that extremes values of the polar radius ρ take place in case:

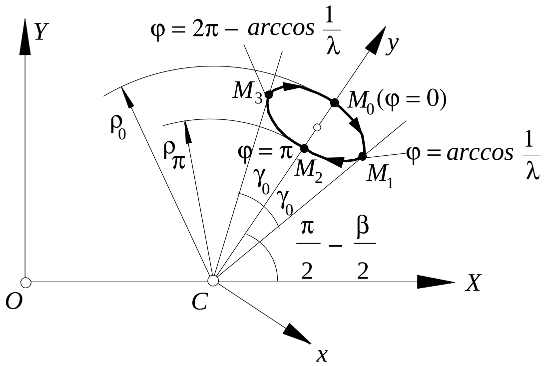


Fig. 3

$$\varphi = 0;\quad \rho_0 = 2\cos\frac{\theta_{\min}+\beta}{2} \tag{20}$$

$$\varphi = \pi;\quad \rho_\pi = 2\cos\frac{\theta_{\max}+\beta}{2} \tag{21}$$

The extremes of the angle γ are obtained for the values of the angle φ for witch we cancel the differentials of the expression (13) that is for the value of the φ inferred of the equation:

$$-\lambda\cos^2\varphi + (1+\lambda^2)\cos\varphi - \lambda = 0 \tag{22}$$

for the case mentioned (λ > 1) we get the values

$$\varphi_1 = \arccos\frac{1}{\lambda};\ \varphi_2 = 2\pi - \arccos\frac{1}{\lambda} \tag{23}$$

and of the relation (13) if we use notation:

$$\gamma_0 = \arcsin\frac{1}{\lambda} \tag{24}$$

it comes out:

$$\gamma_{\min} = -\gamma_0 ; \gamma_{\max} = \gamma_0 \tag{25}$$

Based on the established facts, it comes out the point *M* trajectory is a closed and symmetrical curve (fig.3) limited by the circles with the centre in *C* of radii $\rho_0$, $\rho_\pi$ as well polar radii:

$$\gamma = \gamma_0 ; \gamma = -\gamma_0 \tag{26}$$

If we appeal to polar coordinates then the polar radius $\rho = CM$ and the angle $\gamma$ are given by the relation:

$$\rho = 2\cos\frac{\theta + \beta}{2} \tag{27}$$

$$\gamma = \arcsin\left(\frac{\sin\varphi}{\sqrt{1+\lambda^2 - 2\lambda\cos\varphi}}\right) \tag{28}$$

in case:

$$\theta_{\max} > \frac{\pi}{2} - \frac{\beta}{2} > \theta_{\min} \tag{29}$$

the curve described by point *M* goes through point *C* and in case angle b is negative and carries out the inequality:

$$\beta < -\frac{\theta_{\min} + \theta_{\max}}{2} \tag{30}$$

we obtain the condition:

$$\rho_0 < \rho_\pi \tag{31}$$

**OPTIMAL SYNTHESIS TO GENERATE A TRAJECTORY THAT APPROXIMATES A STRAIGHT LINE SEGMENT**

The description of the curve is made in the sense indicated in fig. 3, precisely: where point *M* starts from $M_0$ for $\varphi = 0$ and moves toward $M_1$ where it moving toward $M_2$ , and it gets there when $\varphi = \pi$ ; it moves then toward $M_3$ where he reaches when $\varphi = 2\pi - \arccos\frac{1}{\lambda}$ and finally move moves toward $M_0$ where it reaches when $\varphi = 2\pi$ .

We propose ourselves to determine Cebâsev mechanism so that a portion of the curve' arch $M_1M_2$ be approximated in a propitious way a straight line segment parallel to the axe *Cx* (fig. 1).

Taking into account the angle $\varphi$ for the point $M_1$ equals $\arccos\frac{1}{\lambda} < \frac{\pi}{2}$ it comes out we can choose values with the interval [90°;180°] for the angle $\varphi$.

For the parameter $\lambda$, *a* chosen to comply with the inequalities (17) , (18) and for a chosen value $\beta$, we determine according to a value $\varphi_i$ of expressed in sexagesimal degrees, in order, the angles $\theta_i$, $\gamma_i$ with the relations (12), (13) and the value $y_i$, with the second relation (14).

Taking into account the index correspondence $\varphi_i = 89^o + i$ we determine the average value $\bar{y}$ with the relations:

$$\bar{y} = \frac{\sum_{i=1}^{90} y_i}{90} \tag{32}$$

Then we calculate the following parameters $\theta_{90}$, $\gamma_{90}$ $x_{90}$ which correspond to the angle $\varphi = 90°$ with the relations:

$$\theta_{90} = 2\arcsin\left(\frac{a}{2}\sqrt{1+\lambda^2}\right)$$
$$\gamma_{90} = \arcsin\left(\frac{1}{\sqrt{1+\lambda^2}}\right) \tag{33}$$
$$x_{90} = 2\cos\frac{\theta_{90}+\beta}{2}\sin\gamma_{90}$$

and is defined the function objective by the relation:

$$S = \frac{1}{x_{90}}\sum_{i=1}^{90}\left|y_i - \bar{y}\right| \tag{34}$$

The constant values λ, *a*, β are determined from the condition that the function objective be minimum.

## THE OPTIMUM SYNTHESIS GENERATING A TRAJECTORY THAT APPROXIMATES A CIRCLE ARCH.

If the synthesis is made with the purpose of getting a mechanism for which point *M* describes a trajectory that approximates a circle arch (fig.4) then we consider an interval of variation of the angle $\varphi \in [0, \tilde{\varphi}]$ and the radius *R* of the arch infers from the equality $\tilde{O}M_0 = \tilde{O}\tilde{M}$ ;

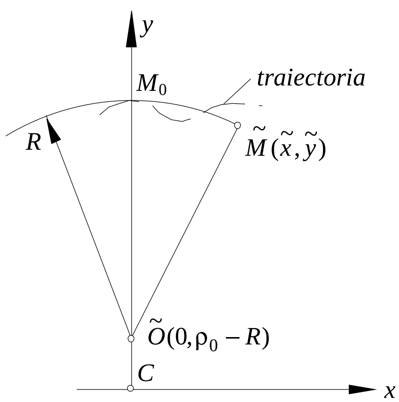


Fig. 4

$$R = \frac{\tilde{\rho}^2 + \rho_0^2 - 2\tilde{y}\rho_0}{2(\rho_0 - \tilde{y})} \tag{35}$$

where the parameters $\tilde{\rho}, \tilde{y}$, are being calculated with relations (4), (12) - (14), the angle φ being equal to $\tilde{\varphi}$.

Further on, we calculate the values $x_i$, $y_i$ with the help of relations (10) where the angle φ takes the values in degrees $i - 1$, *i* taking on his turn, values from 1 to $N = [\tilde{\varphi}]$.

The optimum mechanism is obtained through minimizing the function objective:

$$S = \frac{1}{|R|}\sum_{i=1}^{N}\left|\sqrt{x_i^2 + (y_i + \rho_0 + R)^2} - |R|\right| \tag{36}$$

To generate circle arches on the interval $\varphi \in [90°, 270°]$ we determine the radius of the curvature for a value $\tilde{\varphi} > 90°$ in the relation (35) by replacing $\rho_0$ with $\rho_\pi$ and $x_i$, $y_i$ is calculated for values using degrees proportionally of the angle/ within the interval $[\tilde{\varphi}, 180°]$.

The algorithm and calculus programme and numerical results are displayed in the paper no [5].

## CONCLUSIONS

The method used in the paper allows easily obtaining Cebâsev type mechanisms. Based on the established calculus algorithm, we create a calculus programme by which we can obtain numerical results, fact that is being made in paper [5].

By the established method we carry out the optimum synthesis of the plane quadrilateral mechanisms for which the trajectory of a point on the piston rod's plane approximates a straight line segment or a circle' arch.

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