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# SYNTHESIS OF THE CAMS USING THE JARVIS MARCH

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***Abstract:*** *In this paper we use the Jarvis march to determine the convex cover of the theoretical cam. Based on this algorithm we are able to determine the minimum radius of the base circle of the cam, avoiding the numerical derivation. In the first case one considered the harmonic cam for which the maximum displacement of the flat tappet is known. The second case considers as known the law of motion of the flat tappet. We also highlight the case in which the displacement of the flat tappet takes place on intervals of length different of* $\pi$. *This situation characterizes the Miller-Atkinson cycles.*

**Keywords**: synthesis, harmonic, displacement, Jarvis march

## INTRODUCTION

Considering that the law of motion of the flat tappet is given by $s = s(\varphi)$, then the parametric equations of the cam read [1]

$$x_1 = \frac{\mathrm{d}s}{\mathrm{d}\varphi}\cos\varphi + (r_0 + s)\sin\varphi\,,\; y_1 = -\frac{\mathrm{d}s}{\mathrm{d}\varphi}\sin\varphi + (r_0 + s)\cos\varphi\,, \tag{1}$$

where $r_0$ is the radius of the base circle of the cam.
The great problem of the cams' synthesis is the convexity of the cams' profiles. The expression of the curvature radius [1] is given by

$$R = r_0 + s + \frac{\mathrm{d}^2 s}{\mathrm{d}\varphi^2}\,. \tag{2}$$

In many cases the expression $s = s(\varphi)$ is not analytically known; one has to deal with the numerical derivatives. It is known [2] that the second order derivative is not precisely calculated by numerical derivation.
The avoidance of the self-blocking of the cam mechanism [1] leads to the determination of the minimum radius of the base circle; again, the reader has to calculate a numerical derivative.
In this paper we determine the cam's profile using an algorithm called the Jarvis march. The obtained cam is a convex one. The minimum radius of the base circle is given by the minimum value at which all the points of the cam's theoretical profile are used in the Jarvis march.

## THE JARVIS MARCH

This algorithm is not usually used in the cam's synthesis. Some aspect regarding the Jarvis march are given in [3, 4, 5].

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Let us consider (Figure 1) a planar system of points $A_i(x_i, y_i)$, $i = \overline{1,n}$.
We want to determine the convex cover of this set of points, that is, the minimum convex polygon $P$ that contains the all points $A_i$, $i = \overline{1,n}$, inside it or on its edges.

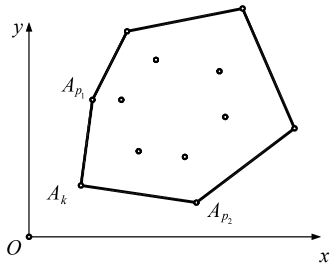



**Figure 1.** The Jarvis march

Obviously, the point $A_k$ for which

$$x_k = \min_{i=\overline{1,n}} x_i \tag{3}$$

is a point of this polygon (Figure. 1).
Starting from this point, one selects the next point $A_p$ so that the all points are situated on the straight line $A_k A_p$ or in the same half-plan defined by this straight line. In Fig. 1, two points $A_{P_1}$ and $A_{P_2}$ could be selected.
The procedure continues in a similar way until no other point can be chosen.

## THE HARMONIC CAM

Let us consider the harmonic cam [1] for which:

$$\rho = \begin{cases} r_0 \text{ for } \psi \in \left[0, \frac{\pi}{2}\right], \\ r_0 + r\cos^2\psi \text{ for } \psi \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right), \\ r_0 \text{ for } \psi \in \left[\frac{3\pi}{2}, 2\pi\right), \end{cases} \tag{4}$$

where $r_0$ is the radius of the base circle, $r$ is the maximum displacement of the flat tappet, $r = 10\,\text{mm}$, while $\rho$ defines the polar radius of the cam.
We will vary the parameter $r_0$ from $r_0 = 15\,\text{mm}$ to $r_0 = 25\,\text{mm}$ and we will mark in Table 1 the number of points used for the convex cover. The angle $\psi$ is varied with a step equal to $0.5^0$; hence, 720 points are used to create the cam's profile. The resulted cam is a convex one if and only if the all 720 points are used in the Jarvis march. In Figure 2 we have drawn the profile of the theoretical cam and its convex cover.

**Table 1.** The harmonic cam

| $r_0$ | Number of points used for the convex cover $n_c$ | Percentage of use of points for the convex cover $p = \frac{n_c}{720} \times 100$ |
|---|---|---|
| 15 | 638 | 88.61 |
| 16 | 648 | 90.00 |
| 17 | 658 | 91.39 |

| $r_0$ | Number of points used for the convex cover $n_c$ | Percentage of use of points for the convex cover $p = \frac{n_c}{720} \times 100$ |
|---|---|---|
| 18 | 672 | 93.33 |
| 19 | 688 | 95.56 |
| 19.99 | 718 | 99.72 |
| 20 | 720 | 100.00 |
| 20.01 | 720 | 100.00 |
| 21 | 720 | 100.00 |
| 22 | 720 | 100.00 |
| 23 | 720 | 100.00 |
| 24 | 720 | 100.00 |
| 25 | 720 | 100.00 |

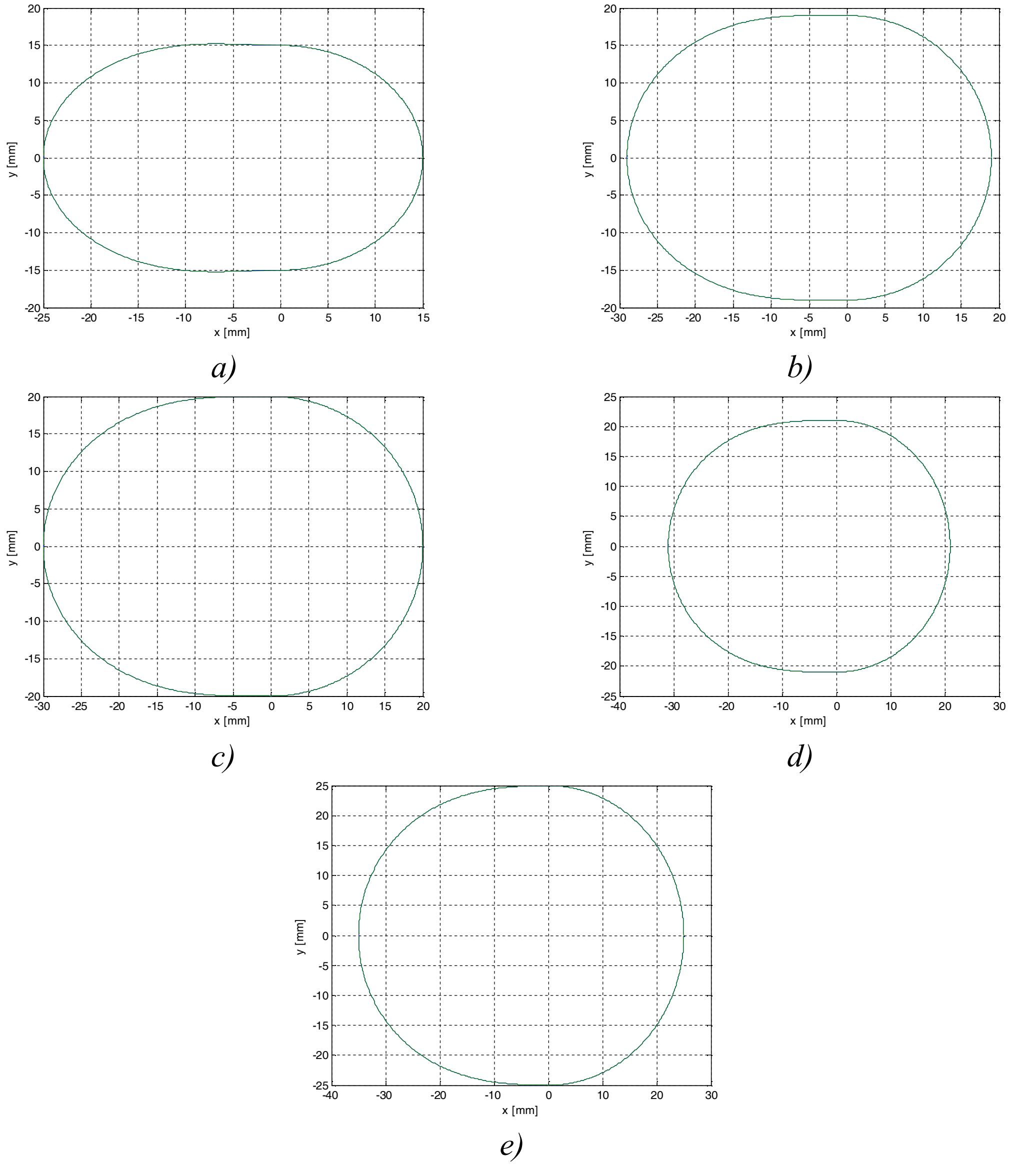



**Figure 2.** The harmonic cam (blue) and its convex cover (green).
*a)* $r_0 = 15\,\text{mm}$; *b)* $r_0 = 19\,\text{mm}$; *c)* $r_0 = 20\,\text{mm}$; *d)* $r_0 = 21\,\text{mm}$; *e)* $r_0 = 25\,\text{mm}$.

One may easily observe that for $r_0 \geq 20\,\text{mm}$ the harmonic cam becomes a convex one. This numerical result confirms the exact value for $r_0$ obtained in an analytical way.

## THE CASE IN WHICH THE DISPLACEMENT OF THE FLAT TAPPET IS KNOWN

We consider the following law for the displacement of the flat tappet (this function is very known in the theory of variations)

$$s(\psi) = \begin{cases} 0 \text{ for } \psi \in \left[0, \frac{\pi}{2}\right], \\ c\left(\psi - \frac{\pi}{2}\right)^2\left(\psi - \frac{3\pi}{2}\right)^2 \text{ for } \psi \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right), \\ 0 \text{ for } \psi \in \left[\frac{3\pi}{2}, 2\pi\right), \end{cases} \tag{5}$$

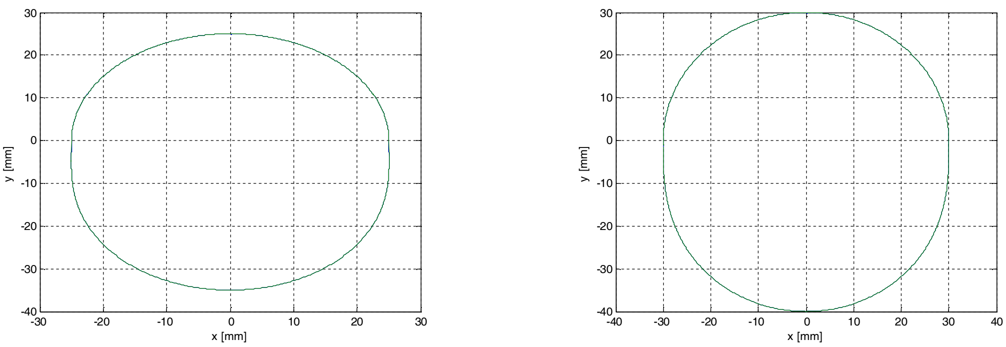
where

$$c = \frac{16r}{\pi^4}. \tag{6}$$

In this way, the maximum value of the function in expression (5) is obtained for $\psi = \pi$, and its value is equal to $r$.
In addition $r = 10\,\text{mm}$, while $r_0$ (the radius of the base circle) takes values between $r_0 = 25\,\text{mm}$ and $r_0 = 40\,\text{mm}$.
Again, the resulted cam is convex if and only if the all 720 points are used. In Figure 3 we represented the profile of the theoretical cam and its convex cover. A table, similar to Table 1, may be also realized.

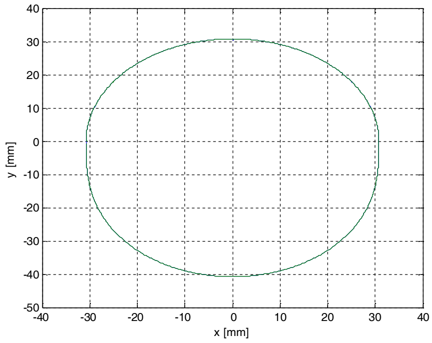
**Table 2.** The cam when the displacement of the flat tappet is known (formula (5))

| $r_0$ | Number of points used for the convex cover $n_c$ | Percentage of use of points for the convex cover $p = \frac{n_c}{720} \times 100$ |
|---|---|---|
| 25 | 686 | 95.28 |
| 27 | 698 | 96.94 |
| 29 | 710 | 98.61 |
| 30 | 714 | 99.17 |
| 30.77 | 718 | 99.72 |
| 30.78 | 720 | 100.00 |
| 31 | 720 | 100.00 |
| 33 | 720 | 100.00 |
| 35 | 720 | 100.00 |

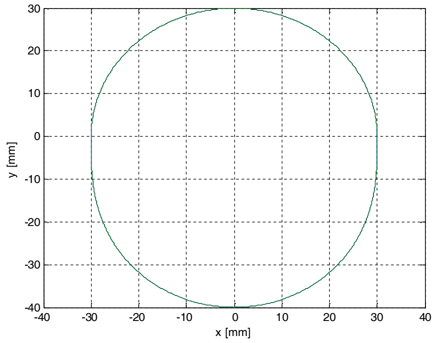



*a)* *b)*

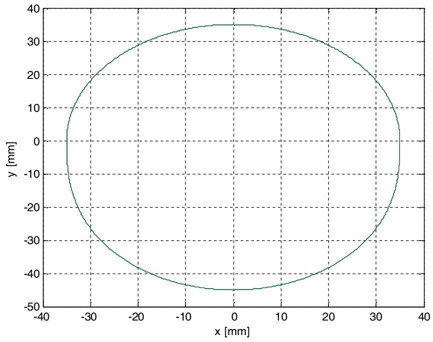
*continued...*




*c)*




*d)*




*e)*

**Figure 3.** The theoretical cam (blue) and its convex cover (green)
*a)* $r_0 = 25\,\text{mm}$; *b)* $r_0 = 30\,\text{mm}$; *c)* $r_0 = 30.77\,\text{mm}$; *d)* $r_0 = 30.78\,\text{mm}$; *e)* $r_0 = 35\,\text{mm}$.

In this situation, the cam is convex if the radius $r_0$ of the base circle fulfills the condition $r_0 \geq 30.78\,\text{mm}$. One may vary the law for the displacement of the flat tappet, modifying the formula (5) as follows (used, for instance, for the Miller-Atkinson cycles).

$$s(\psi) = \begin{cases} 0 \text{ for } \psi \in \left[0, \dfrac{\pi}{2}\right], \\ c_1\left(\psi - \dfrac{\pi}{2}\right)^2 (\psi - \pi)^2 \text{ for } \psi \in \left(\dfrac{\pi}{2}, \pi\right), \\ 0 \text{ for } \psi \in [\pi, 2\pi), \end{cases} \tag{6}$$

where

$$c_1 = \frac{256 r}{\pi^4} \tag{7}$$

or

$$s(\psi) = \begin{cases} 0 \text{ for } \psi \in \left[0, \dfrac{\pi}{2}\right], \\ c_2\left(\psi - \dfrac{\pi}{2}\right)^2 (\psi - 2\pi)^2 \text{ for } \psi \in \left(\dfrac{\pi}{2}, 2\pi\right), \end{cases} \tag{8}$$

with

$$c_2 = \frac{256 r}{81 \pi^4} \tag{9}$$

The corresponding data are given in Tables 3 and 4.

**Table 3.** The cam when the displacement of the flat tappet is known (formula (6))

| $r_0$ | Number of points used for the convex cover $n_c$ | Percentage of use of points for the convex cover $p = \frac{n_c}{720} \times 100$ |
|---|---|---|
| 20 | 421 | 58.47 |
| 40 | 446 | 61.94 |
| 60 | 466 | 64.72 |
| 80 | 484 | 67.22 |
| 100 | 498 | 69.17 |
| 125 | 514 | 71.39 |
| 150 | 527 | 73.19 |
| 200 | 550 | 76.39 |
| 300 | 586 | 81.39 |

**Table 4.** The cam when the displacement of the flat tappet is known (formula (8))

| $r_0$ | Number of points used for the convex cover $n_c$ | Percentage of use of points for the convex cover $p = \frac{n_c}{720} \times 100$ |
|---|---|---|
| 20 | 672 | 93.33 |
| 40 | 686 | 95.28 |
| 60 | 692 | 96.11 |
| 80 | 696 | 96.67 |
| 100 | 698 | 96.94 |
| 125 | 701 | 97.36 |
| 150 | 702 | 97.50 |
| 200 | 705 | 97.92 |
| 300 | 708 | 98.33 |

## CONCLUSIONS

In our paper we described a new approach in the cams' synthesis using the algorithm called the Jarvis march. The advantages of this approach are:
– the obtained cam is always a convex one, no matter which input data we used;
– one obtains the minimum radius of the base circle for which the theoretical cam becomes a convex one.
In our future work, we will implement this approach on a real distribution mechanism.

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