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# The collapse modes classification of tubes with different shapes of the cross-section

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**Abstract.** In this paper, the authors present the collapse modes classification of tubes under axial loads. This topic is sensitive because pollution requirements are becoming more stringent and the emissions value of $CO_2$ should be reduced. Therefore, to achieve this, modern vehicles must have a lower weight. The condition that a vehicle has a lower weight and high performance is that materials should have a lower weight and at the same time a high degree of protection. In case of a collision, the dimensions of the cockpit shouldn't change, hence the energy is consumed by the controlled deformation of the sacrificial structures. These sacrificial structures are defined by tubes/structures with closed profiles.

**Keywords:** axial crushing; circular tube; collapse mode; classification

## 1. Introduction

As in any field, safety is the most important attribute in the automotive domain. It's known that the actual government regulation requires manufacturers from the automotive domain to develop innovative systems and conceive to the improvement of structural crashworthiness to reducing mortality and injury of the occupants. On the other hand, another sensitive problem is to minimize fuel consumption (reduce the $CO_2$ emissions value) and environmental sustainability pushes the structures lighter and lighter. As a result, thin-walled structures, are used as an energy absorber in vehicle structures, but at the same time, the thin-walled structures should have a high degree of strength [1], [2],[3],[4].

Nevertheless, the researchers have analyzed the behavior of the thin-walled tube under axial load [5],[6],[7],[8]. The maximum force value represents the maximum value from the first fold and is noted with $F_{max}$. From the specialized literature three main types of collapse mode are identified (concertina, diamond, mixed mode) [9],[10],[11],[12]. These modes are mainly influenced by material properties and at the same time by the reports $D/t$ and $L/D$ where $D, t, L$ are diameter, thickness, and length [3].

A lot of studies have approached the topic of thin-walled structures [13],[14],[15]. The graphic below shows a summary classification of thin-walled structures, materials and collapse mode of structures.

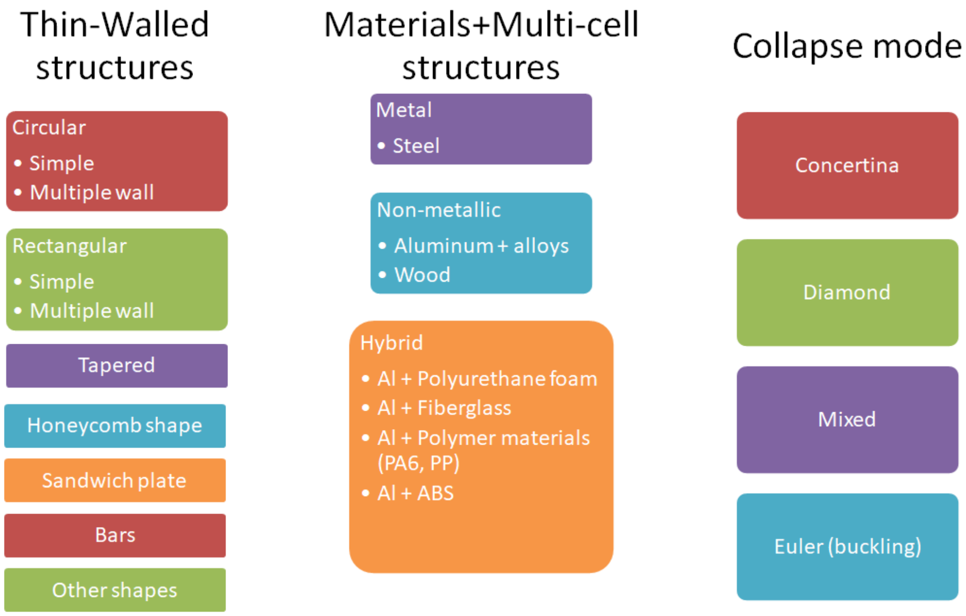


**Figure 1.** Summary classification of thin-walled structures, materials, collapse mode

## 2. Theoretical background

The foundations of the subject presented above were laid in 1959 by J. M. Alexander, in a paper regarding the design of nuclear reactors having vertical fuel channels [5]. To absorb the energy resulting from falling weights, energy absorber devices should be identified, must of the time the shapes of the structures are like a thin-walled tube, which has the capacity axial crushing under load. There are two real ways of structure collapse, concertina, and diamond resulting in a combination of them namely the mixed mode.

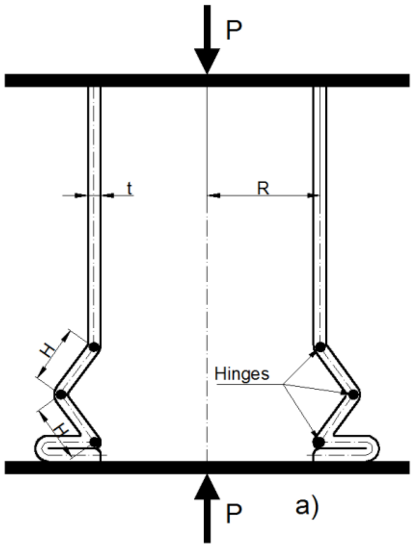


**Figure 2.** Axi-symmetric collapse mechanism

W. Abramowicz performs a lot of experimental tests on tubes of circular cross-section made from steel, under static and dynamic axial crushing [16]. Three types of collapse modes are known:

1) Axi-symmetric mode (concertina) – the folds are similar;
2) Non-symmetric mode (diamond) – the folds have a different number of corners;
3) Mixed mode (concertina + diamond) – a combination of two previous modes.

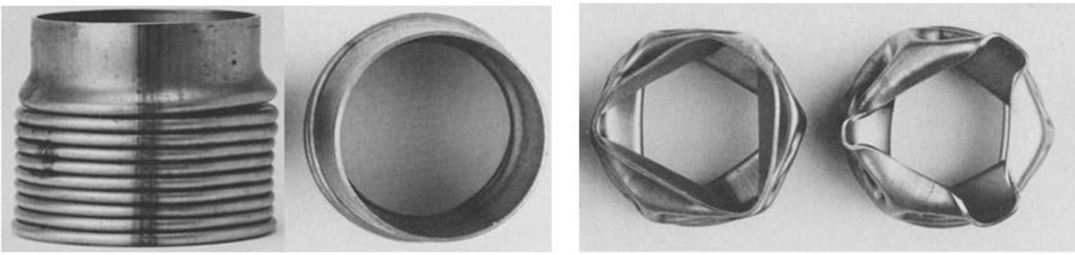

**Figure 3.** Concertina deformation mode vs. Diamond deformation mode

The researches regarding this subject are focused, also on the nonmetallic components and the range of the materials begins to expand a lot (polyurethane foam, wood, paper, plastic) [4]. The axial crushing of circular tube was firstly analyzed by Alexander [5] and found to be an excellent mechanism for energy absorption [9]. The thin-walled circular cross-section structures have the most energy absorption capacity and the most mean force among all investigated sections [17]:

$$\frac{D}{t} > 20 \tag{1}$$

where:
$D$ – mean diameter of the circular structure;
$t$ – wall thickness of the circular structure.

C. R. Calladine continued the research and studies the design of columns of mild steel and columns of aluminum alloys thus showing that Perry-Robertson's formula can provide a good approximation of the buckling loads of long and thin columns [18]. Compared with previous studies in 1983 W. Abramowicz takes into account the properties of the material and collapse effect of the material [19].

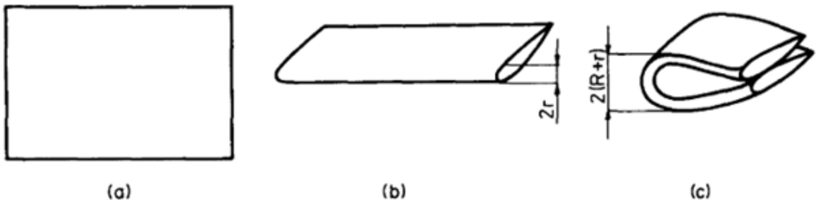


**Figure 4.** Subsequent bendings of initially flat metal sheet

Regarding the figure from Alexander's study, which is a generic axial crushing model of circular and square tubes, can be stated that the energy is:

$$E_{fold} = E_b + E_m \tag{2}$$

where:
$E_{fold}$ – Fold energy;
$E_b$ – Bending energy consumed along the lines of plastic hinges;
$E_m$ – Membrane energy.
Mean crushing force $F_m$, developed during axial compression can be equal with:

$$F_m = \frac{E_{fold}}{2H} \quad \rightarrow \quad F_m \cdot 2H = E_b + E_m \tag{3}$$

For a good agreement, a new formula is developed for an evaluation of the mean crushing force take into account the enhancing coefficient [20]:

$$F_m \cdot 2H \cdot \kappa = E_b + E_m \tag{4}$$

In the work of Alexander [5] the stress state displayed by the collapsed section was defined considering von Mises criterior of yielding and the material's initial yield stress $\sigma_y$:

$$\sigma_0 = \frac{2}{\sqrt{3}} \cdot \sigma_y \tag{5}$$

With respect to the data of a stress-strain curve, the flow stress can be defined as the average of initial yield stress $\sigma_y$ and ultimate stress $\sigma_u$:

$$\sigma_0 = \frac{\sigma_y + \sigma_u}{2} \tag{6}$$

where**:**

$\sigma_y$– initial yield stress;
$\sigma_u$– ultimate stress.
Membrane energy for a circular cross-section tube is:

$$F_m = 22.27 \cdot M_o \cdot \sqrt{\frac{D}{t}} \tag{7}$$

where $M_0 = 1/4 \cdot \sigma_0 \cdot t^2$
For the circular cross-section, the first evaluation of the mean crushing force was performed by Alexander [5], followed by a solution proposed by Wierzbicki [3]:

$$E_b = 4 \cdot \pi \cdot M_o \cdot (\pi \cdot R + H) \tag{8}$$

The energy dissipated in the three joints along the deformation of a fold can be written as:

$$E_b = 4 \cdot \pi \cdot M_o \cdot (\pi \cdot R + H) \tag{9}$$

The diamond mode is also studied which in some cases causes an inclination of a column which could lead to a collapse in the sense of Euler (buckling) [21],[22]. In 1992, a new model of the progressive crushing of circular tubes was developed in which an active zone of plastic deformations contains two folds named in the specialty literature "Super Folding Elements" and is very suitable for prismatic structures and also for multi-corner structures [3],[23].

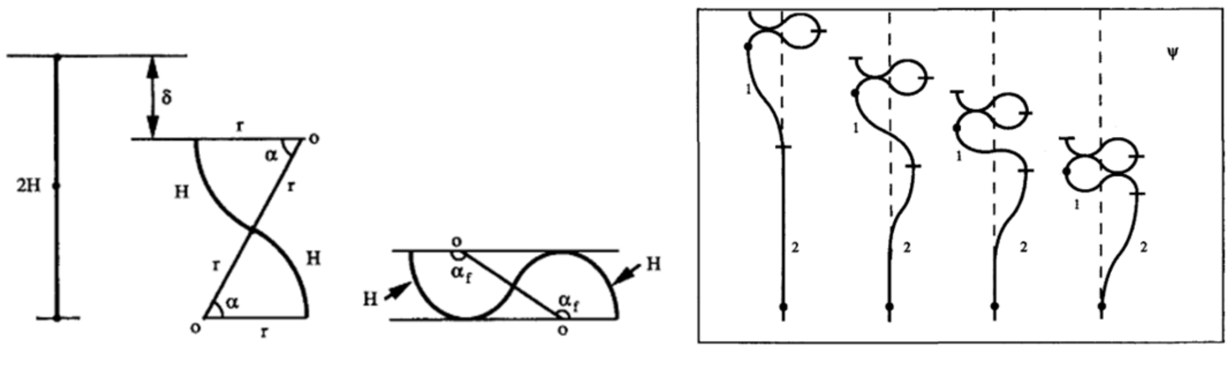


**Figure 5.** Undeformed, partially deformed, fully crushed individual SFE and subsequent deformation stages of the two-element model

Thin-walled structures are analyzed by the finite element method, where it's observed that the shell of the structure under axial load is sensitive to velocity and also to the mass of the impactor. Structures with various geometric shapes have been used, filled with a wide variety of materials and foams. The following table presents the geometric shapes and materials used in experimental test [24].

**Table 1.** Geometric shapes of the structures and materials used

| **Geometric shapes** | **Materials** |
|---|---|
| - Circular tubes | - Aluminum + alloys |
| - Frusta shape | - Stainless steel |
| - Square tubes | - Wood |
| - Steel bars | - Polyurethane foam |
| - Sandwich plates | - Polymer materials (PA6, PP) |
| - Honeycomb shape | - Fiberglass |
| - Multi-cell structures | - Other materials |
| - Other shape | |

## 3. Composite structures

Since 2006, the researchers turned their attention to the multi-cell structures. Multi-cell section was split into three areas, namely the corner part, the crisscross part, and the T-shape part [9].

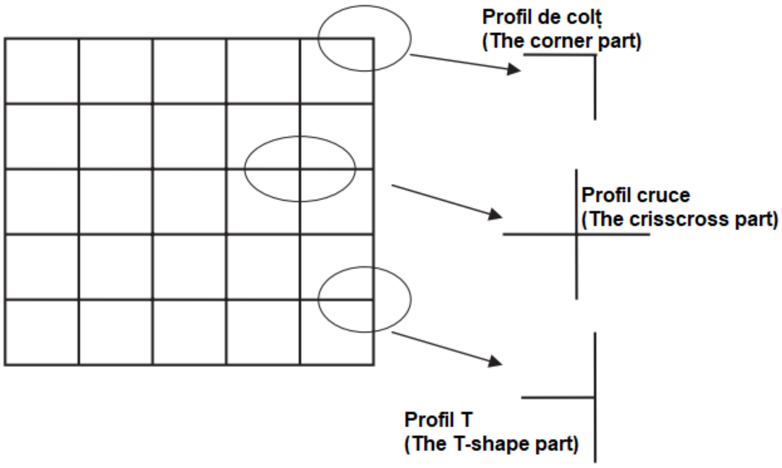


**Figure 6.** Illustration of the division of multi-cell section

For the figure presented above (Fig. 6), the energy dissipated in the three parts can be presented in the below table:

| Energy dissipated | | |
|---|---|---|
| Corner part | Crisscross part | T-shape part |
| $M_{corner} = 4 \cdot M_0 \cdot H^2/t$ | $M_{crisscross} = 16 \cdot M_0 \cdot H^2/t$ | $M_{T-shape} = 8 \cdot M_0 \cdot H^2/t$ |

**Table 2.** The energy dissipated in the corner part, crisscross part and T-shape part

The energy dissipated by membrane deformation is [9]:

$$W_{membrane} = N_C \cdot M_{corner} + N_0 \cdot M_{crisscross} + N_T \cdot M_{T-shape} \quad (10)$$

The Mean Crushing Force for a multi-cell section is:

$$F_m = \sigma_0 \cdot t \cdot \sqrt{(N_C + 4 \cdot N_0 + 2 \cdot N_T) \cdot \pi \cdot L_C \cdot t} \quad (11)$$

where:
$N_C, N_0, N_T$ – denote the number of corner, crisscross and T-shape;

$\sigma_0$ – flow stress of the material;
$M_0$ – fully plastic bending moment of the flange;
H – half-length of the fold;
t – wall thickness;
$L_C \cdot t$ – material area of the cross-section.

For multi-cellular structures with $x\ N$ , the number of the profiles will be calculated as follows:

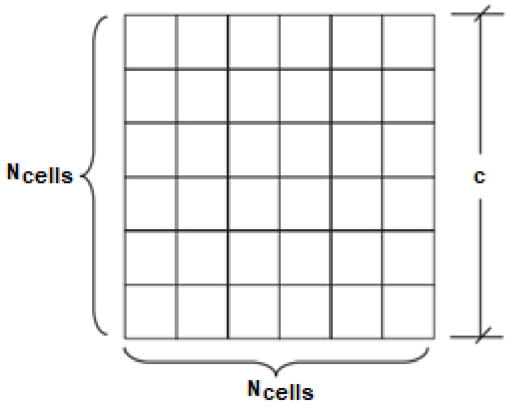


**Figure 7.** Sketch map of an $N\ x\ N$ cell section

**Table 3.** Multi-cellular structure description

| | |
|---|---|
| Number of corner | $N_C = 4$ |
| Number of crisscross | $N_0 = (N-1)^2$ |
| Number of T-shape | $N_T = 4 \cdot (N-1)$ |
| Wall thickness | $t = \frac{A_m}{2 \cdot (N+1) \cdot c}$ |

The Mean Crushing Force for a multi-cellular is:

$$F_m = \sqrt{\pi} \cdot \sigma_0 \cdot \frac{N \cdot A_m^{3/2}}{(N+1) \cdot a} \tag{12}$$

Two years later, the studies showed that multi-cellular metal structures proved to be more efficient than single-cell structures. The Mean Crushing Force for multi-cellular square structures can be [25]:
square single cell:

$$F_{S1} = 6.88 \cdot \sigma_0 \cdot C^{1/2} \cdot t^{3/2} \tag{13}$$

double cell:

$$F_{S2} = 9.89 \cdot \sigma_0 \cdot C^{1/2} \cdot t^{3/2} \tag{14}$$

triple cell:

$$F_{S3} = 12.94 \cdot \sigma_0 \cdot C^{1/2} \cdot t^{3/2} \tag{15}$$

quadruple cell:

$$F_{S4} = 14.18 \cdot \sigma_0 \cdot C^{1/2} \cdot t^{3/2} \tag{16}$$

four square cells in corner:

$$F_{S5} = 19.30 \cdot \sigma_0 \cdot C^{1/2} \cdot t^{3/2} \quad (17)$$

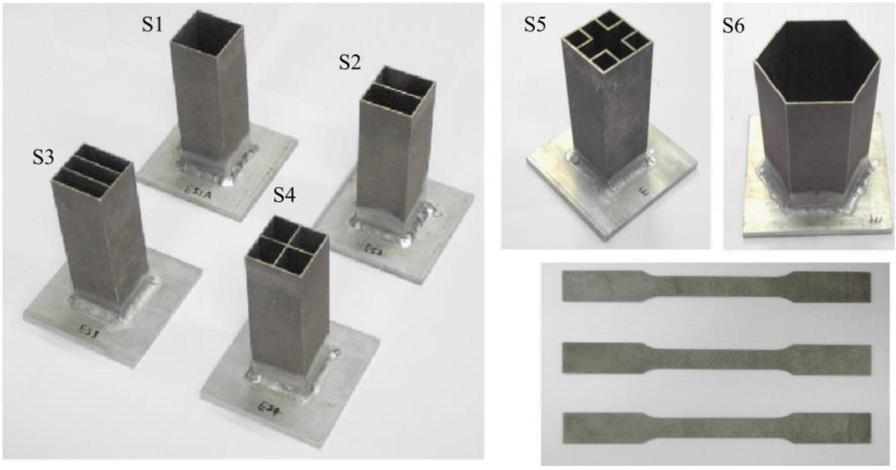


**Figure 8.** Multi-cell and tensile specimens

In 2015, the researchers studies simple tubes with circular cross-section and also tube with inserts, the structures being subjected under axial load [20].

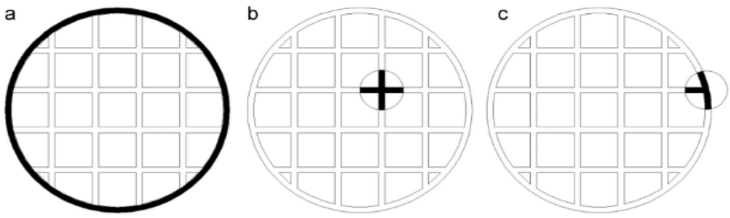


**Figure 9.** Typical sections of the structure. a) Outer tube; b) Crisscross; c) T-shape joint

The energy dissipated in the structure with circular cross-section:
bending energy for outer tube:

$$E_{b,tube} = 4 \cdot \pi^2 \cdot M_0 \cdot R \quad (18)$$

bending energy for insert:

$$E_{b,insert} = 2 \cdot \pi \cdot M_0 \cdot L_i \quad (19)$$

membrane energy crisscross:

$$E_{m,crisscross} = 16 \cdot M_0 \cdot H^2 \cdot \frac{1}{t} \quad (20)$$

membrane energy T-shape:

$$E_{m,T-shape} = 4 \cdot M_0 \cdot H^2 \cdot \frac{1}{t} \quad (21)$$

In the case of structures with polygonal and star-shaped cross-section, it's possible to exist an extensional mode to crushing mode. For each case analyzed, an analytical solution was implemented in extensional mode using the formula with $\kappa = 0.75$:

$$F_m = \frac{1}{\kappa} \cdot \sigma_0 \cdot t \cdot \sqrt{(\pi - \theta) \cdot \pi \cdot L_e \cdot t} \tag{22}$$

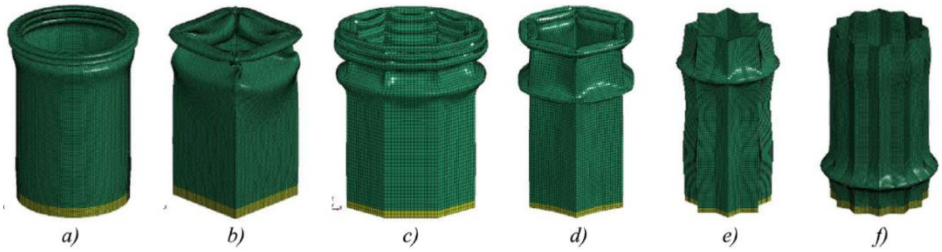


**Figure 10.** Types of structures. a) circular b) rectangle c) octagon d) e) f) star

For multi-walled structures, the general shape of the bending stiffness (BS) for the specific composite beam can be calculated using the following expression [26]:

$$BS = \sum_i^n E_i \cdot I_i \tag{23}$$

where:
$n$ – number of walls
$E$ – elasticity modulus [MPa]
$I$ – enertia moment $[mm^4]$
In 2014, in the specialty literature appear a new term „hybrid component", the structure is made from a steel tube having inside a structure made from polyamide and fiberglass. Based on the studies performed on the experimental, theoretical and numerical results of various tubes under axial load, in the last years, a set of KPI performance indicators has been proposed to evaluate and compare the performance of the energy absorption.

The main indicators of the KPI are [27]:

ESR – effective stroke ratio:

$$ESR = \frac{S_{ef}}{L} \tag{24}$$

NLC – Non-dimensional load-carrying capacity:

$$NLC = \frac{F_m}{M_0} \tag{25}$$

SEA – Specific energy absorption capacity:

$$SEA = \frac{F_m}{\rho A} \cdot \frac{S_{ef}}{L} \tag{26}$$

EEA – Effectiveness of energy absorption:

$$EEA = \frac{F_m}{AY} \cdot \frac{S_{ef}}{L} \tag{27}$$

SLC – Stableness of load-carrying capacity:

$$SLC = \frac{F_m}{F_p} \tag{28}$$

where:
$S_{ef}$ – effective stroke;
$\rho AL = G$ – total mass of the tube;
$F_p$ – Initial Peak Force.

## 4. Conclusions

With time different methods have been studied that can meet two important conditions. First of all the structure should be very resistant and on the other hand, the structure should be slight in case of potential impact. Therefore, in case of impact, kinetic energy must be consumed. The collapse structure can be achieved by applying an axial impact load or an oblique impact load. It's known that the dimensions of the cockpit shouldn't deform, and the acceleration should be minimal so that the health of passengers is not affected. Thus the sacrificial structures represented by tubes with closed profiles are investigated. When the energy is absorbed, the structure undergoes modifications and in the specialized literature are presented three types of collapse mode:

- Concertina mode
- Diamond mode
- Mixed mode

Also, these structures are defined as tubes with closed profiles, the notion of the tube being generically used. The technology has evolved and the profiles become more and more complex to meet the various requirements. For maximum absorption of kinetic energy, the sacrificial structures must take overload avoiding buckling.
Starting from the standard geometry of a structure, it is observed that energy is absorbed by deformation. This process is intensively studied due to the avoidance to use huge structures. For a structure to absorb a big quantity of energy, the parameters that can be changed are the diameter and the thickness of structure.
For this reason, the research was directed towards multi-cells structures, but checking the performance parameters we observe that increasing the diameter and thickness does not necessarily offer an increase the performance (bending and stretching).The mean crushing force is directly proportional to the diameter and thickness of the structure. In the case of multi-cell structures to keep the weight of the part, a part of the wall thickness of the structure is distributed inside it.
Compared to the simple structures where we had a profile and a stretching mechanism, in case of multi-cells structures we discuss about mechanisms presented above (T-shape part, crisscross part or corner part).

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